Unsteadiness in porous media with Lattice Boltzmann Method.

Simon Marié
associate professor

Laboratoire DynFluid, Conservatoire National des Arts et Métiers.

December 11th 2023

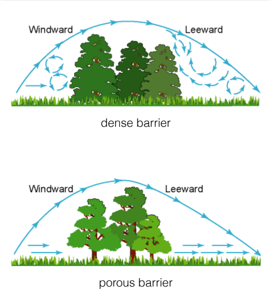
Understanding instabilities in porous media

Porous materials and porous surfaces are present in a wide variety of flows: biological flows, geological flows, industrial flows. . .

Understanding instabilities in porous media

Porous materials and porous surfaces are present in a wide variety of flows: biological flows, geological flows, industrial flows. . .

Foliage density on trees

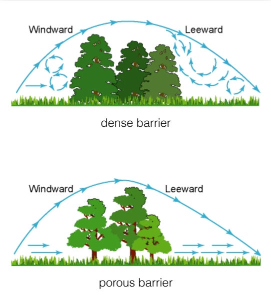


Velvet-like surfaces on bird wing

Understanding instabilities in porous media

Porous materials and porous surfaces are present in a wide variety of flows: biological flows, geological flows, industrial flows. . .

Foliage density on trees



Velvet-like surfaces on bird wing

Steady state of these kind of flows have been widely studied but unsteadiness can occurs and are more complex to understand...

Understanding instabilities in porous media

Numerical simulations of Porous media can be tricky and complex:

Understanding instabilities in porous media

Numerical simulations of Porous media can be tricky and complex:

- ✓ Diversity of length scales.
- ✓ Large time scale.
- ✓ Complex geometry.
- ✓ Possibly led by very different phenomenon: Diffusion, convection, mass and heat transfer. . .

Understanding instabilities in porous media

Numerical simulations of Porous media can be tricky and complex:

- ✓ Diversity of length scales.
- ✓ Large time scale.
- ✓ Complex geometry.
- ✓ Possibly led by very different phenomenon: Diffusion, convection, mass and heat transfer. . .

Mesoscopic simulations can bring some insight in the understanding of unsteadiness in porous media...

1 Porous media in LBM

- Models and limitations
- Important parameters

2 Diffusive unsteadiness

- Miscible Multicomponent LBM
- Saffman-Taylor with 2 components
- Saffman-Taylor with 3 components

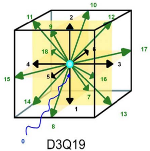
3 Convective unsteadiness

- About the Brinkmann Boundary layers
- Flow over a porous sphere
- Validation at a moderate Reynolds number, $Re = 200$
- Influence of the Da number on the flow at $Re = 300$

Lattice Boltzmann model

D3Q19 with MRT collision model, 2nd order equilibrium and forcing term.

$$\frac{\partial g(\mathbf{c},\mathbf{x},t)}{\partial t} + \underbrace{c_i \frac{\partial g(\mathbf{c},\mathbf{x},t)}{\partial x_i}}_{streaming\ (advection)} + \underbrace{\boldsymbol{F_B} \frac{\partial g(\mathbf{c},\mathbf{x},t)}{\partial \mathbf{c}}}_{source\ term} = \underbrace{\left(\frac{\partial g}{\partial t}\right)_{\text{coll}}}_{collision} \qquad (1)$$



Collision	$\mathbf{m}^{coll}$	$=$	$\mathbf{m} - \mathcal{D}(\mathbf{m} - \mathbf{m}^{eq}) + \underbrace{(1 - \frac{dt}{2})\mathcal{D}\mathcal{M}\mathbf{S}(\boldsymbol{F_B})}_{source\ term}$
Streaming	$\mathbf{g}(\mathbf{x},t)$	$=$	$\mathcal{M}^{-1}\mathbf{m}^{coll}(\mathbf{x} - \mathbf{c}_\alpha dt, t - dt)$
Forcing term	S_α	$=$	$\omega_\alpha \left[\frac{\mathbf{c}_\alpha - \mathbf{u}}{c_0^2} + \frac{(\mathbf{c}_\alpha \cdot \mathbf{u})\mathbf{c}_\alpha}{c_0^4}\right] \cdot \mathbf{F_p}$
Brinkmann Force	$\mathbf{F_p}$	$=$	$- \frac{\phi}{ReDa}\chi\mathbf{u}$

The Brinkman penalization method

Among all the possibilities to impose porous conditions in LBM (GLBM, partial bounce-back. . .), the Brinkman penalization method is local and allows to easily define global parameters such as porosity (Φ) and permeability (K).

The Darcy law is then recovered with the diffusion term $\tilde{\mu}\Phi\Delta u$:

$$\nabla p = \tilde{\mu}\Delta u - \frac{\mu\Phi}{K}u$$

where $\tilde{\mu}$ denotes the Brinkman effective viscosity and verifies the following relation with respect to the dynamic viscosity of the fluid : $\tilde{\mu}/\mu = 1/\Phi$. Therefore, if one assumes a porosity close to one ($\Phi \sim 1$), one has $\tilde{\mu} \sim \mu$.

The Brinkman penalization method

Among all the possibilities to impose porous conditions in LBM (GLBM, partial bounce-back...), the Brinkman penalization method is local and allows to easily define global parameters such as porosity (Φ) and permeability (K).

The Darcy law is then recovered with the diffusion term $\tilde{\mu}\Phi\Delta u$:

$$\nabla p = \tilde{\mu}\Delta u - \frac{\mu\Phi}{K}u$$

where $\tilde{\mu}$ denotes the Brinkman effective viscosity and verifies the following relation with respect to the dynamic viscosity of the fluid : $\tilde{\mu}/\mu = 1/\Phi$. Therefore, if one assumes a porosity close to one ($\Phi \sim 1$), one has $\tilde{\mu} \sim \mu$.

Thus, the porosity will be fixed to a constant in all the computations:

$$\phi = 0.98$$

$\rightarrow$ Possibility to relax this condition by doing volume averaging.

Important parameters

Scales:

- ✓ Mass diffusion: $\mathcal{D}$
- ✓ Momentum diffusion: ν
- ✓ Convection scale: U
- ✓ Length scale: L

Physics:

- ✓ Peclet number for mass diffusion: $Pe = \dfrac{UL}{\mathcal{D}}$
- ✓ Darcy number : $Da = \dfrac{K}{L^2}$
- ✓ Reynolds number for fluid flows: $Re = \dfrac{UL}{\nu}$

Particular values of these dimensionless numbers can lead to unsteadiness in porous media.

Multicomponent mixture model

- ✓ One distribution per component.
- ✓ One relaxation parameters for each distribution
- ✓ Global inter-collision modeled with Kerkhof term.
- ✓ Applicable to any collision model.

Multicomponent LBM:

$$f_\alpha^m\left(\mathbf{x}+\mathbf{e}_\alpha\delta_t, t+\delta_t\right) = f_\alpha^m\left(\mathbf{x}, t\right) - \frac{1}{\tau_m}\left[f_\alpha^m\left(\mathbf{x}, t\right) - f_\alpha^{m(eq)}\left(\mathbf{x}, t\right)\right] + S_\alpha^m\left(\mathbf{x}, t\right)$$

Forcing term:

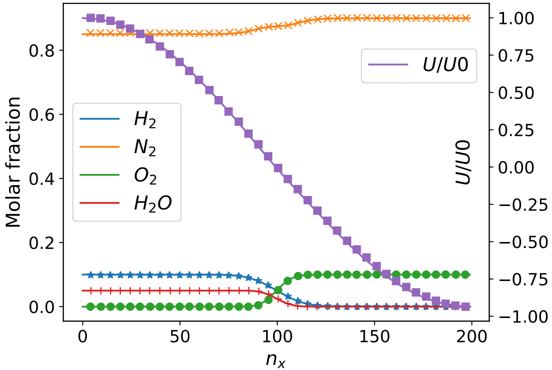
$$S_\alpha^m = \left(1-\frac{\delta_t}{2\tau_m}\right)\omega_\alpha\left[\frac{\mathbf{e}_\alpha-\mathbf{u}_m}{c_s^2} + \frac{(\mathbf{e}_\alpha\cdot\mathbf{u}_m)\mathbf{e}_\alpha}{c_s^4}\right]\cdot\mathcal{F}_m$$

Diffusion term: Kerkhof & Geboers

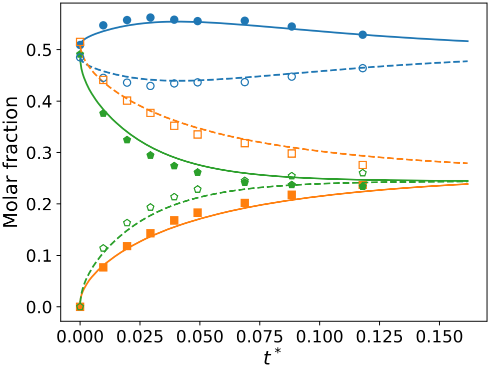
$$\mathcal{F}_m = -p\sum_{n=1}^{N}\frac{x_m x_n}{\mathcal{D}_{mn}}(\mathbf{u}_m-\mathbf{u}_n)i$$

Numerical validation on diffusive and convective cases.[1]

Opposed jets



Loschmidt tube



[1] L. Vienne, S. Marié, and F. Grasso (2019). "Lattice Boltzmann model for miscible gases: a forcing term approach". In: *Physical Review E* 100 (2), p. 023309

Viscous fingering

Viscous fingering:

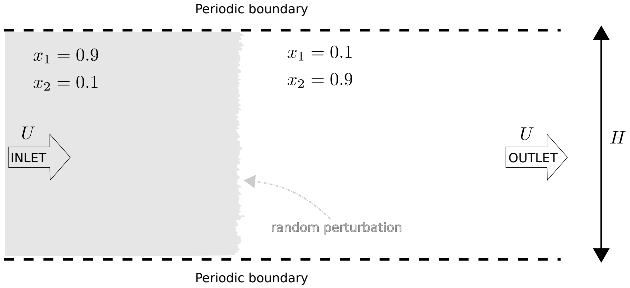
- ✓ Mixing in porous media is particularly difficult due to the absence of inertia
- ✓ but it plays a key role in carbon sequestration, mantle convection, microfluidic device . . .

This interfacial instability occurs when a less viscous fluid displaces a more viscous fluid in a porous medium.

Viscous fingering (*Homsy 1987*).

Viscous fingering

Initial conditions



Fixed permeability in the whole domain

$$R_e D_a \sim 10^{-6}$$

Dimensionless numbers:

$$R = \ln\left(\mu_{0,2}/\mu_{0,1}\right) = 3$$

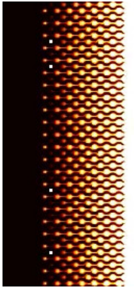
$$P_e = \frac{UH}{\mathcal{D}_{12}} = [500 \ldots 16000]$$

Initial conditions

$$f_\alpha^m(t=0) = f_\alpha^{m(eq)}(\rho_m, u_{x,m} = U, u_{y,m} = 0)$$

Saffman-Taylor instability

Calculated porous (Pore-Scale with Bounce-Back) :



Saffman-Taylor instability

Modeled porous (Brinkman Force).

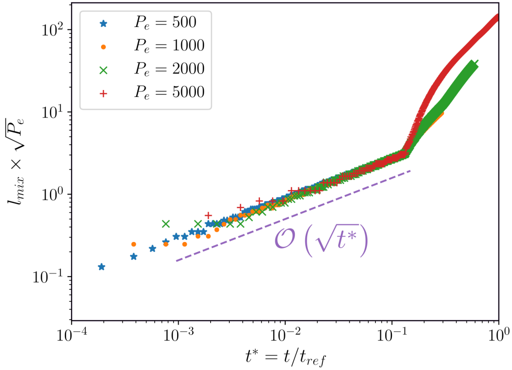
Viscous fingering: Mixing length

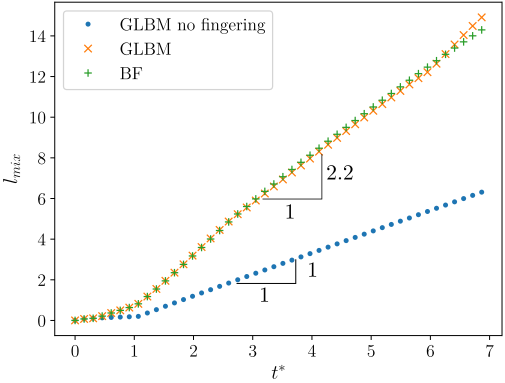
Mixing length

$$l_{mix}(t) = ||\bar{X}_{0.899}(t) - \bar{X}_{0.101}(t)||$$

✓ Diffusion dominated regime for $t^* < 0.1$

✓ Advection dominated regime for $t^* > 1$





Viscous fingering: Early times

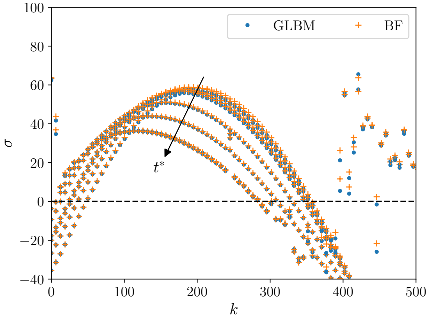
Assuming a perturbation:

$$x'_m(\mathbf{x},t) = x'_m(\mathbf{x}) \exp(\sigma t)$$

Growth rate of the perturbation σ:

$$x'_m(\mathbf{x},t) = x^0_m(\mathbf{x},t) - x_m(\mathbf{x},t)$$
$$\hat{x}(x,k,t) = \mathrm{FFT}_y\left(x'_m(\mathbf{x},t)\right)$$
$$a(k,t) = ||\hat{x}(x,k,t)||_2 = \sqrt{\int_0^{n_x} \hat{x}\cdot\hat{x}\,dx}$$
$$\sigma(k,t) = \frac{d\ln\left(a(k,t)\right)}{dt}$$

x^0_m: base state $\Longleftrightarrow$ non-perturbed simulation.



Dispersion curves for $R = 3$, $P_e = 2000$

- ✓ Growth rate decreases in time.
- ✓ Most dangerous, threshold and cutoff wave numbers are reduced as the instability progresses.

Viscous fingering: Influence of the Péclet number $P_e = \frac{UH}{\mathcal{D}_{12}}$

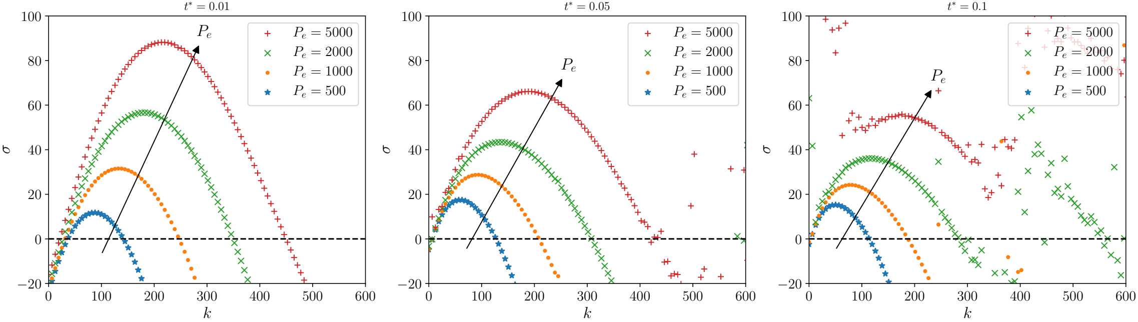
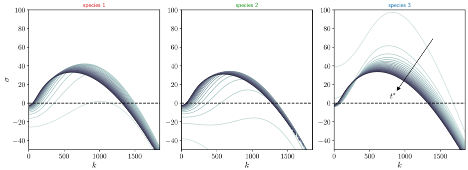


Figure: Dispersion curves for $R = 3$ at $t^* = 0.01,\ 0.05,\ 0.1$ with different Péclet values.

- ✓ High Péclet numbers lead to a more intense instability.
- ✓ The range of unstable wave numbers and the growth rate increase with P_e.
- ✓ The Péclet number influences the transition from linear to non-linear interactions.

Saffman-Taylor with ternary mixture.



Simulation with 3 components

Molar fractions	Fluid 1	Fluid 2
x_1 (R)	0.1	0.45
x_2 (G)	0.45	0.1
x_3 (B)	0.45	0.45
RGB color		

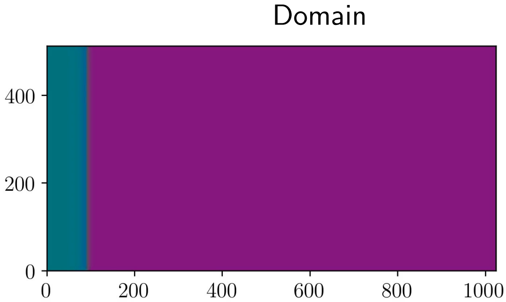
First mixture is more viscous → No instability.

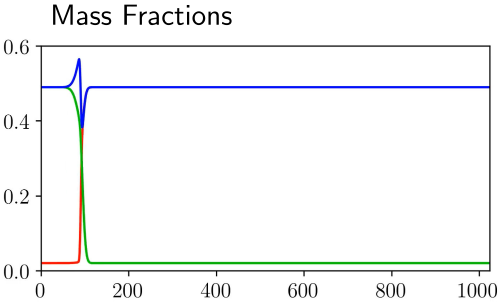
BUT

1. Back-diffusion → Mass transfer from less viscous fluid in the first mixture
2. First mixture becomes less viscous → Diffusive instability appears,
3. Convective régime occurs lately → rise of the growth rate.

Saffman-Taylor with ternary mixture.[2]

3 components instability





[2] L. Vienne and S. Marié (2021). "A Lattice Boltzmann study of miscible viscous fingering for binary and ternary mixtures". In: *Physical Review F* 6 (5), p. 053904

Summary

- ✓ LBM model for multi-component flow using an inter-molecular friction force.
- ✓ The instability is achieved with pore-scale or subscale simulations with Brinkmann forcing scheme.
- ✓ Mixing length: 2 regimes are identified (diffusive then convective).
- ✓ High Péclet numbers result in a more intense instability.
- ✓ **3 species**: initially stable configuration becomes unstable because of back diffusion.

future work

- ✓ Impact of larger Darcy number on multicomponent mixing.
- ✓ Modeling turbulent mixing in porous media (Higher Reynolds)

Wakes behind porous media

Objectives:

- ✓ Influence of permeability on the flow dynamics ?
- ✓ Transitions and bifurcations of fluid-porous flows

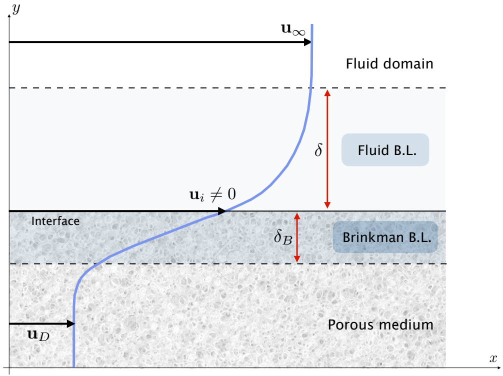
Wakes behind porous media

Objectives:

- ✓ Influence of permeability on the flow dynamics ?
- ✓ Transitions and bifurcations of fluid-porous flows

Problematic:

- ✓ Porous model: **Brinkman penalization** method.
- ✓ Prediction of transition region between porous and fluid region ?
- ✓ Estimation of u_i ?
- ✓ Estimation of δ_B ?

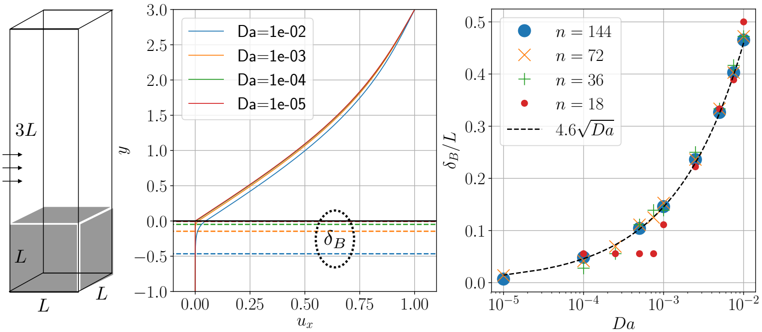


Capturing the boundary layers

Capturing the thickness δ_B of the Brinkman boundary layer

<u>Definition of δ_B</u> : height with respect to the porous-fluid interface where the difference of the tangential velocity u_x with the Darcy velocity u_{D_x} is reduced to 1% of the corresponding value at the interface u_{i_x}.

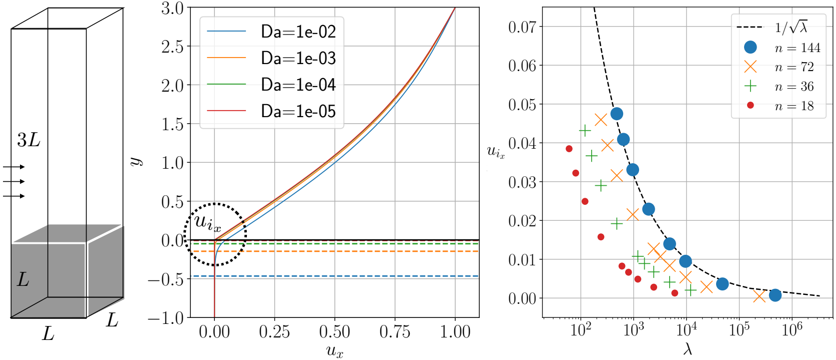
Breugem et al. (2005) give an analytical estimation : $\delta_B \approx 4.6 \cdot \sqrt{k/\Phi}$



Capturing the boundary layers

Capturing the interfacial velocity u_i

The analysis carried out by Ueda & Kida (2021) on the nonlinear penalized Brinkman Navier-Stokes equations states that the tangential and normal slip velocities at the interface verify: $u_{i_x} \xrightarrow[\lambda\to\infty]{} \mathcal{O}(1/\sqrt{\lambda})$ and $u_{i_y} \xrightarrow[\lambda\to\infty]{} \mathcal{O}(1/\lambda)$ with $\lambda = \Phi/(ReDa)$.



Capturing the boundary layers

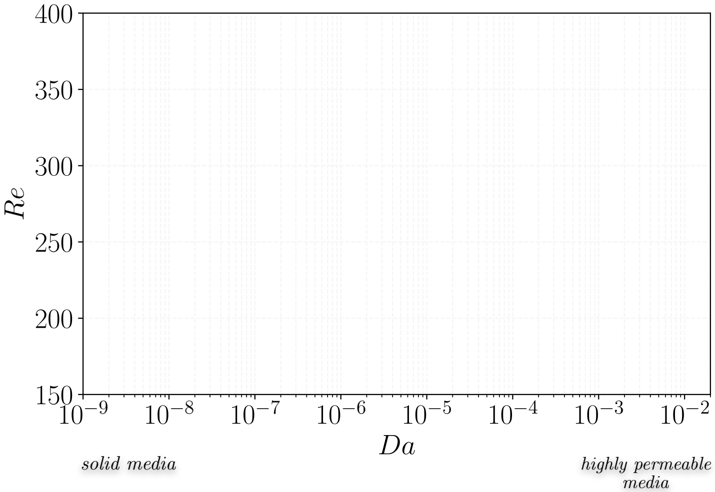
Conclusions

A correct evaluation of the interfacial velocity u_i with a Brinkman penalization approach requires :

- ✓ a sufficient resolution in the porous region.
- ✓ a good representation of the **Brinkman boundary layer**
  $\longrightarrow$ however, the grid-resolution constraint is relaxed when increasing the permeability (i.e. when $Da \nearrow$)

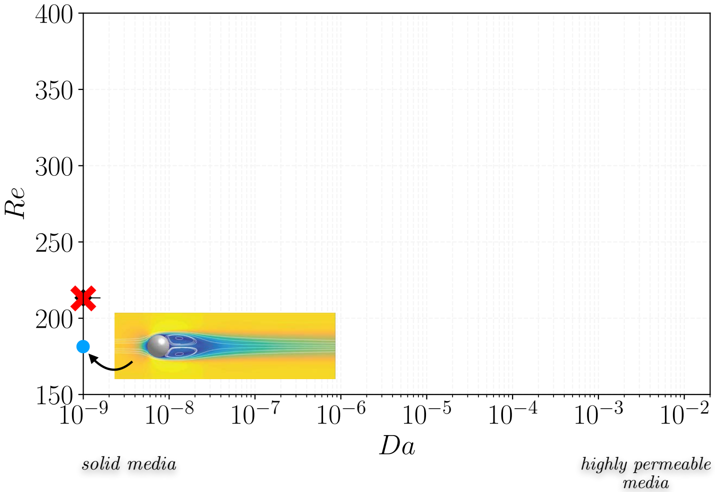
Flow over a porous sphere

Motivations



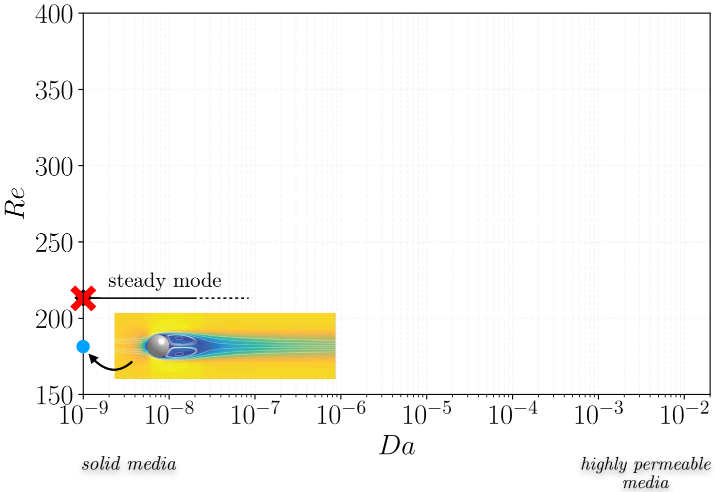
Flow over a porous sphere

Motivations



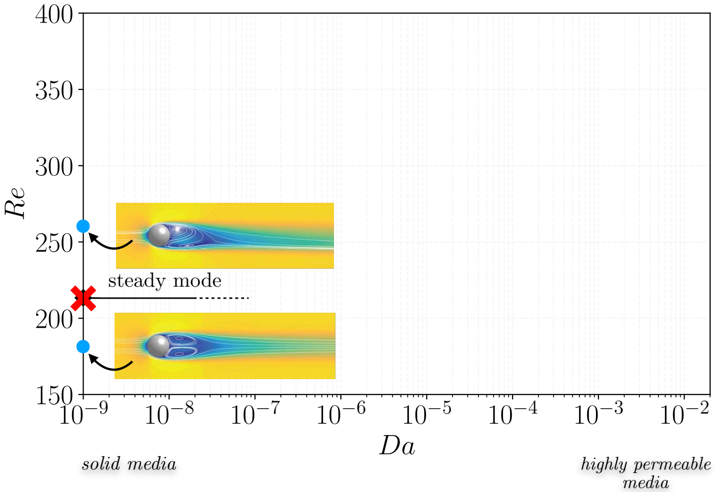
Flow over a porous sphere

Motivations



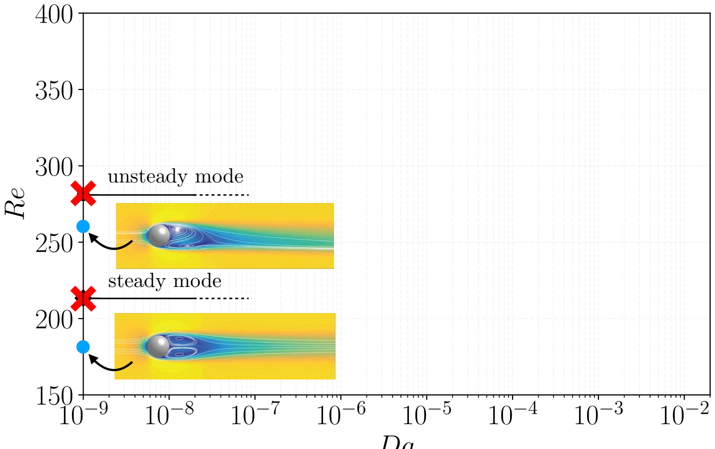
Flow over a porous sphere

Motivations



Flow over a porous sphere

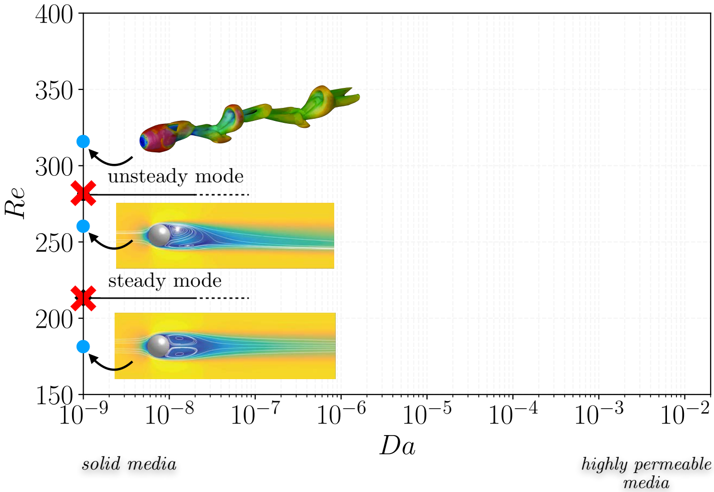
Motivations



solid media *highly permeable media*

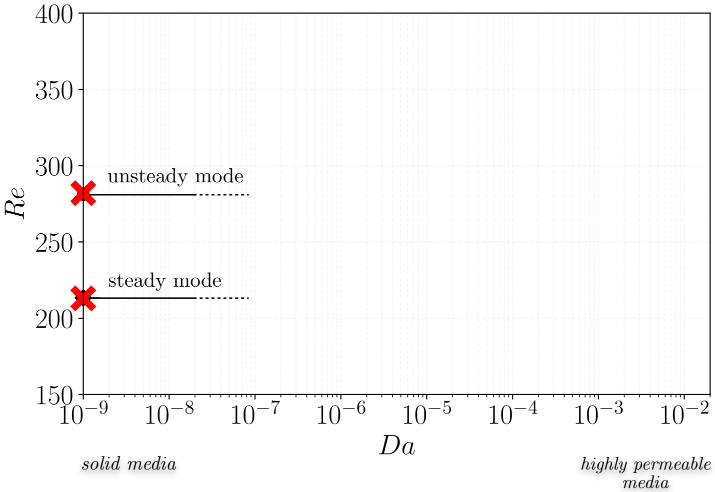
Flow over a porous sphere

Motivations



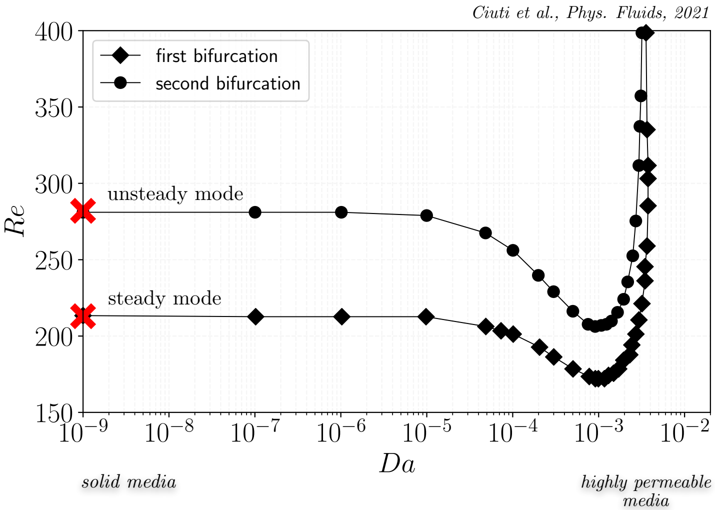
Flow over a porous sphere

Motivations



Flow over a porous sphere

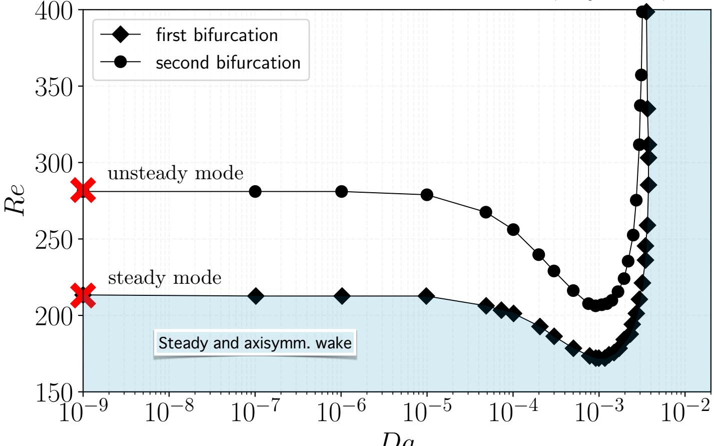
Motivations



Flow over a porous sphere

Motivations

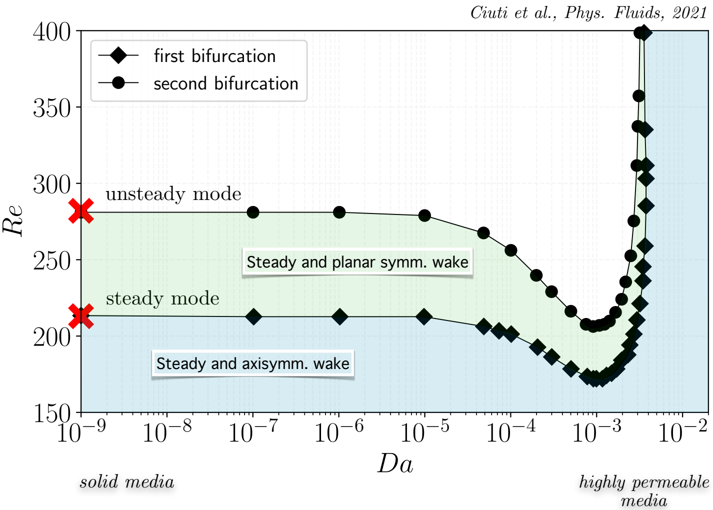
Ciuti et al., Phys. Fluids, 2021



solid media *highly permeable media*

Flow over a porous sphere

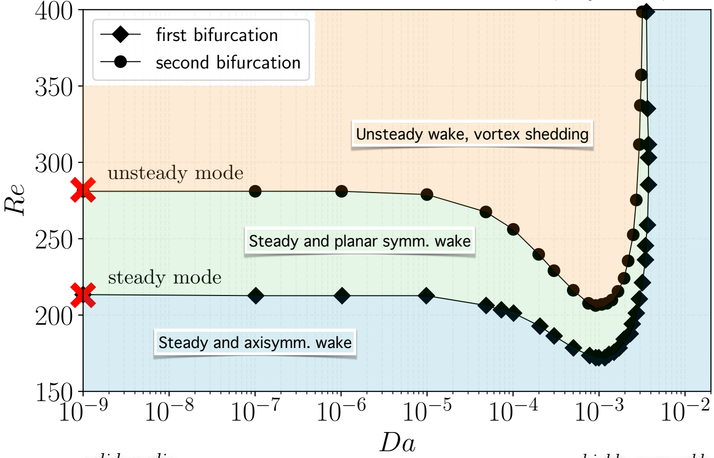
Motivations



Flow over a porous sphere

Motivations

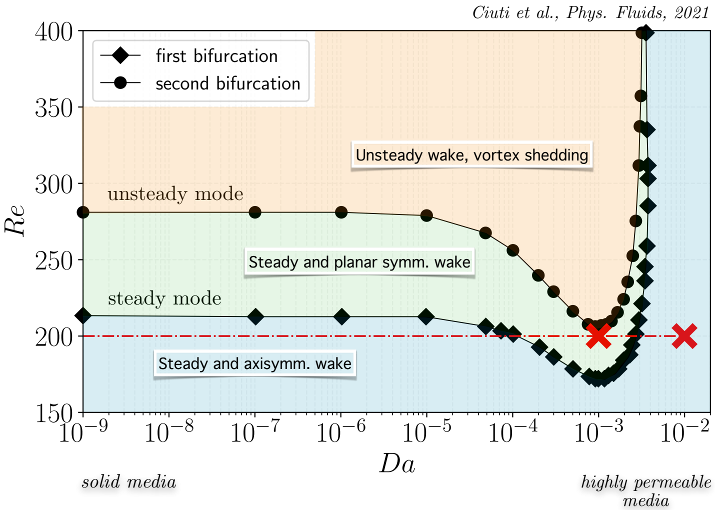
Ciuti et al., Phys. Fluids, 2021



solid media *highly permeable media*

Flow over a porous sphere

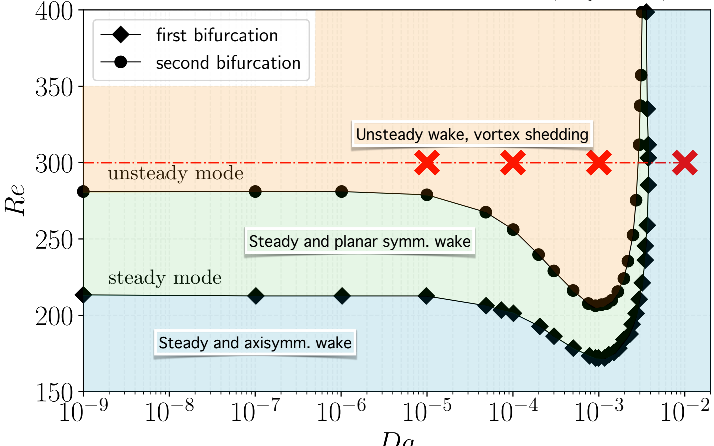
Motivations



Flow over a porous sphere

Motivations

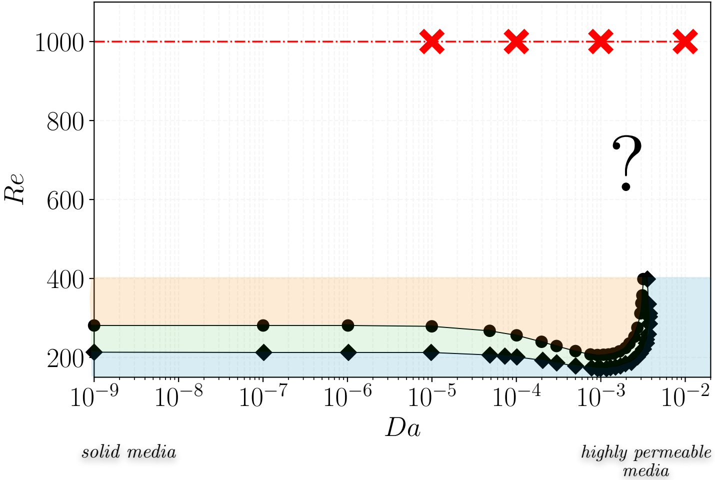
Ciuti et al., Phys. Fluids, 2021



solid media | *highly permeable media*

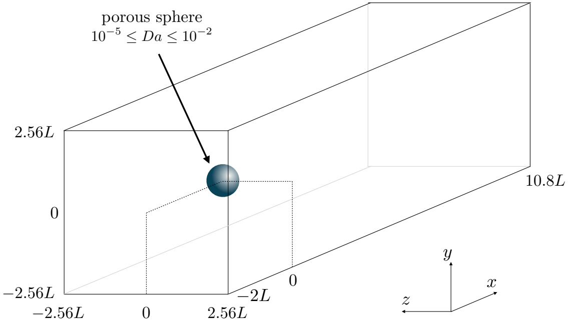
Flow over a porous sphere

Motivations



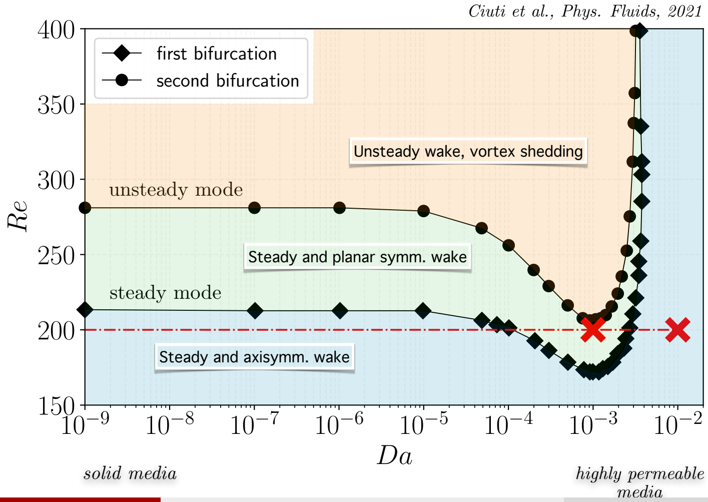
Flow over a porous sphere

Numerical setup



Grids	$N_x \times N_y \times N_z$	n (cells/sphere diameter)
G1	$80 \times 32 \times 32$	6.25
G2	$160 \times 64 \times 64$	12.5
G3	$320 \times 128 \times 128$	25
G4	$640 \times 256 \times 256$	50
G5	$1024 \times 410 \times 410$	80
G6	$1280 \times 512 \times 512$	100

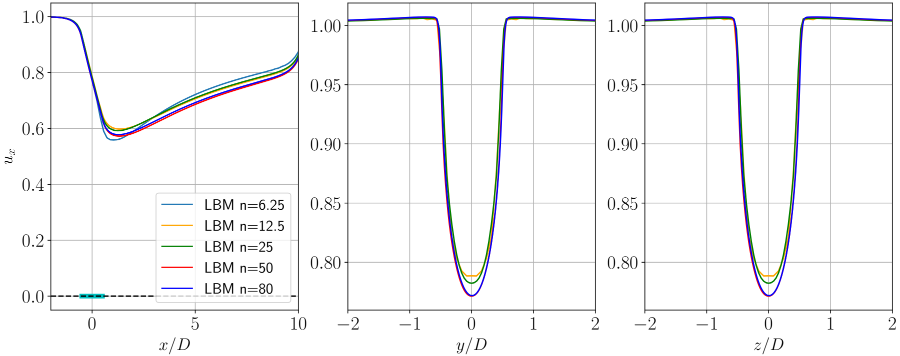
Validation at a moderate Reynolds number, $Re = 200$



Validation at a moderate Reynolds number, $Re = 200$

Porous sphere at $Da = 10^{-2}$: axisymmetric and steady flow

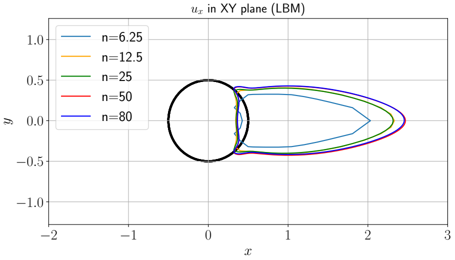
Grid convergence of the streamwise velocity profiles :



Validation at a moderate Reynolds number, $Re = 200$

Porous sphere at $Da = 10^{-2}$: axisymmetric and steady flow

◇ Grid convergence of the $u_x = 0$ contour :

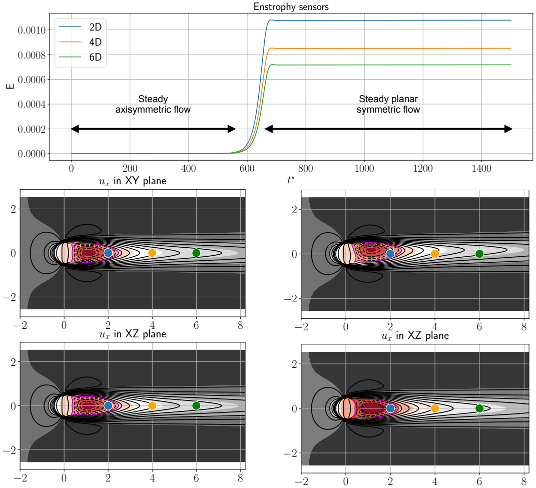


◇ Grid convergence of the L_r and X_r lengths :

Present method	X_r	L_r
$n = 6.25$	-	1.51
$n = 12.5$	-0.1	1.92
$n = 25$	-0.14	1.92
$n = 50$	-0.12	2.08
$n = 80$	-0.125	2.05
Ciuti et al.	-0.2	2.25

Validation at a moderate Reynolds number, $Re = 200$

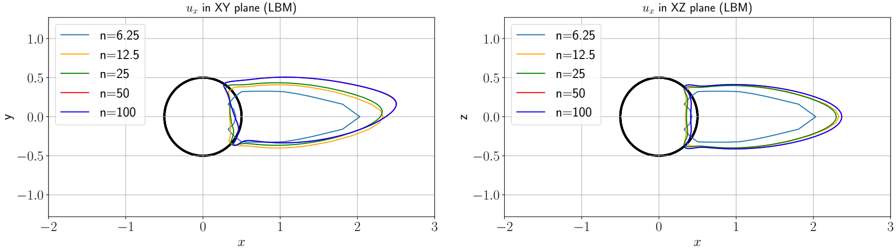
Porous sphere at $Da = 10^{-3}$: steady planar-symmetric flow



Validation at a moderate Reynolds number, $Re = 200$

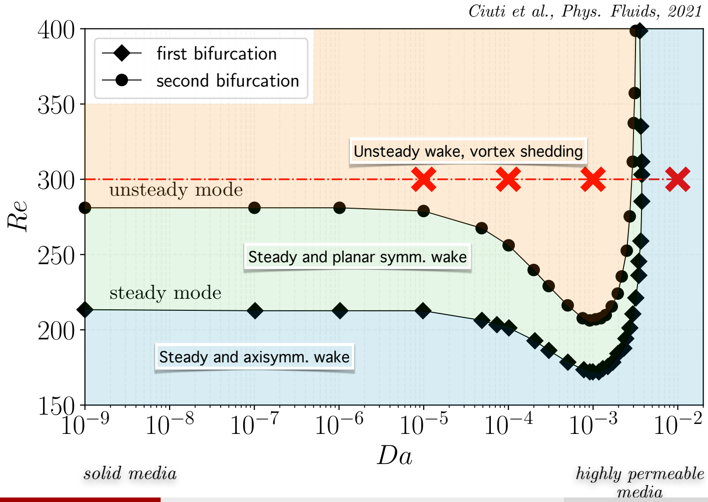
Porous sphere at $Da = 10^{-3}$: steady planar-symmetric flow

Steady and planar-symmetric flow :



Grid convergence of the $u_x = 0$ contour

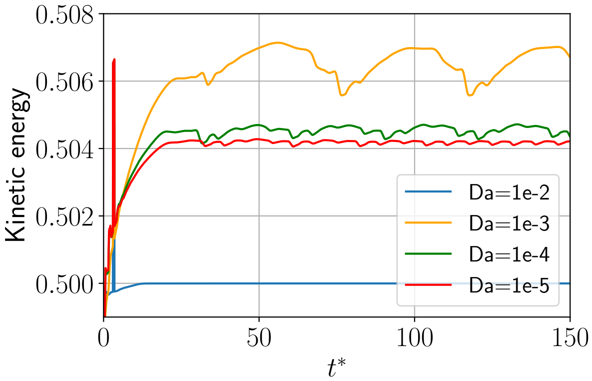
Influence of the Da number on the flow at $Re = 300$



Influence of the *Da* number on the flow at *Re* = 300

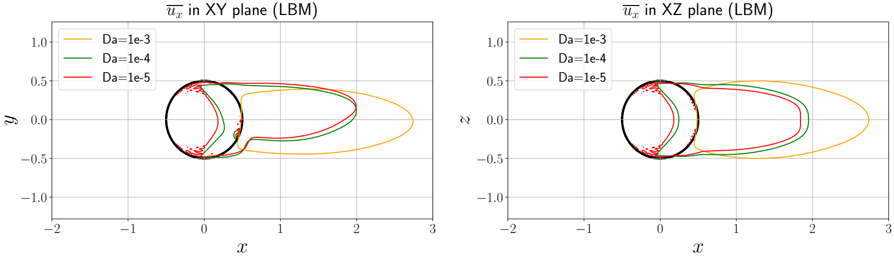
Steadiness

- ✓ $Da = 10^{-2}$: **steady** flow
- ✓ $Da = 10^{-3}$ and $Da = 10^{-4}$: pseudo-periodic **unsteady** flow
- ✓ $Da = 10^{-5}$: tends to the time-periodic **unsteady** flow of a solid sphere



Influence of the *Da* number on the flow at $Re = 300$

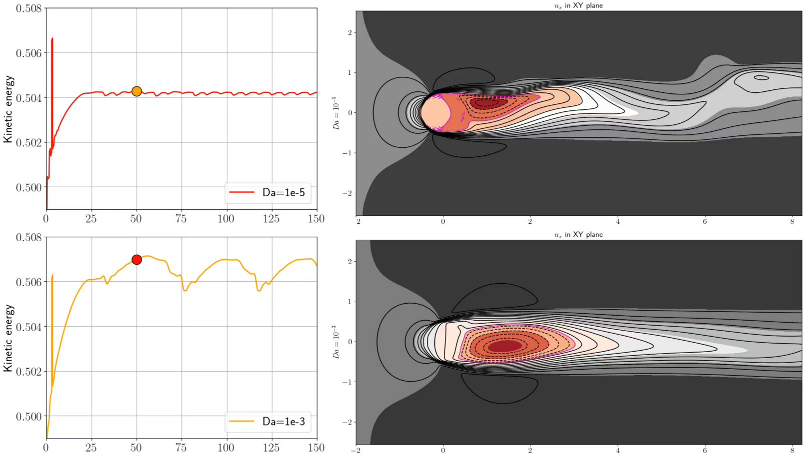
Spacial structures of the mean flow



Isocontour of the mean velocity field $\overline{u}_x$ (computed on the $t^* \in [50, 125]$ time-range)

Influence of the Da number on the flow at $Re = 300$.[3]

Spacial structures of the instantaneous flow with $Da = 10^{-5}$ and $Da = 10^{-3}$

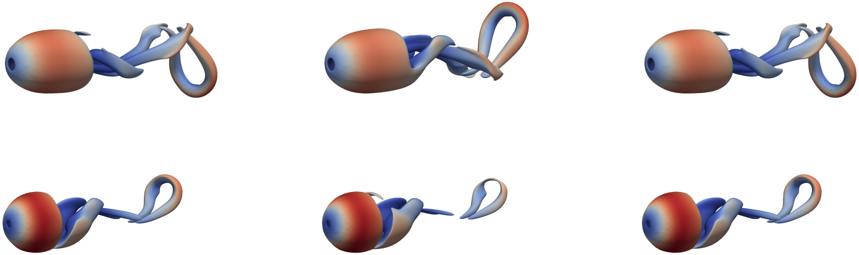


[3] Mimeau Chloé et al. (Aug. 2023). "Wake Prediction in 3D Porous–Fluid Flows: A Numerical Study Using a Brinkman Penalization LBM Approach". In: Flow, Turbulence and Combustion. ISSN: 1573-1987.

Influence of the Da number on the flow at $Re = 300$.[4]

Spacial structures of the instantaneous flow with $Da = 10^{-5}$ and $Da = 10^{-3}$

$t = t_0$ | $t = t_0 + T$ | $t = t_0 + 2T$

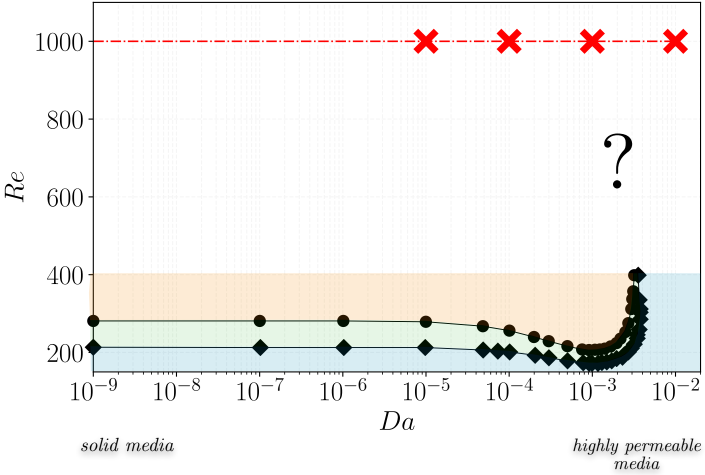


$Da = 10^{-3}$

$Da = 10^{-5}$

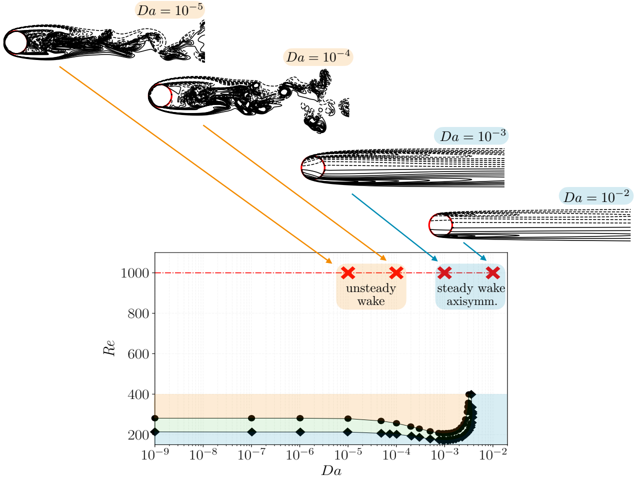
[4] Mimeau Chloé et al. (Aug. 2023). "Wake Prediction in 3D Porous–Fluid Flows: A Numerical Study Using a Brinkman Penalization LBM Approach". In: *Flow, Turbulence and Combustion*. ISSN: 1573-1987. DOI: 10.1007/s10494-023-00471-w. URL: http://dx.doi.org/10.1007/s10494-023-00471-w

Flow past a porous sphere at $Re = 1000$



Flow past a porous sphere at $Re = 1000$

- ✓ Both 10^{-3} and 10^{-2} Darcy yield to steady axisymmetric flows.
- ✓ Critical Darcy seems to be lower for $Re = 1000$
- ✓ Unsteadiness have different patterns for $Da = 10^{-4}$ and $Da = 10^{-5}$



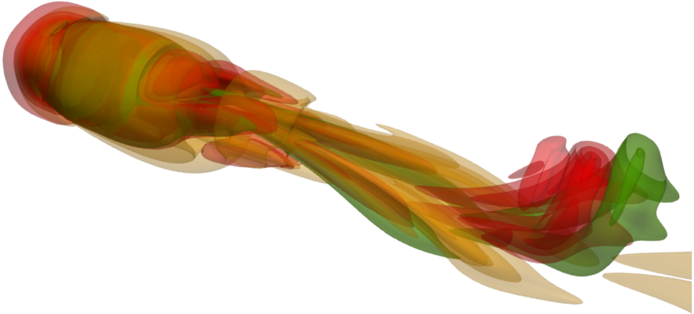
Summary

- ✓ The Brinkman forcing term in LBM allows to correctly capture the expected interfacial velocity u_i and the thickness of the Brinkman B.L. δ_B
- ✓ able to correctly predict transitions in flows past a porous bluff-body, if space resolution s enough ($\sim$ 50 point in the sphre diameter)
- ✓ The unsteadiness exhibits different patterns when Darcy number is decresased.
- ✓ Higher Reynolds number seems to decrease the critical value of the Darcy number.

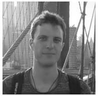
Perspectives:

- ✓ Simulations at higher *Re* numbers : Deeper investigate the contribution of the quadratic Forchheimer term.
- ✓ Designing a porous-wall model for transitional and turbulent flows in porous media (on going PhD)

Thank you for your attention



Collaborations:



Lucien Vienne
CNRS - LMFA

Chloé Mimeau
CNAM - M2N

Léo Roussel
CNAM - DynFLuid