Consistent filtering for the lattice Boltzmann computation of aerodynamic noise.

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**Introduction**

- BGK Lattice Boltzmann Method is known to be a low-dissipative scheme (Marié, Ricot, and Sagaut 2009\(^1\)).
- Consequently, numerical stability is rarely provided for large Reynolds numbers.
- Numerous methods have been proposed to increase numerical stability (MRT, artificial viscosity, entropic formulation...).
- Lots of them induce a global over-dissipation and could damp some low-amplitude physical modes (acoustics).
- Stabilization procedure can drastically increase computational cost.

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Introduction

- Brogi et al. 2017\(^2\) introduce a regularisation step after collision to damp unphysical modes and manage to keep low enough dissipation of acoustic waves.

- Gendre et al. 2017\(^3\) propose to use a two-relaxation time with enhanced interpolation for transition of resolution.

- Here some adaptive filtering technique consistent with LBM stencil and acoustic propagation is proposed for BGK and MRT models.

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1 Numerical models
   - Basic idea of adaptive filtering
   - BGK implementation
   - MRT implementation

2 Aerodynamic validation
   - 3D Taylor-Green-Vortex
   - Taylor-Green results

3 Acoustic Validation
   - Sound radiated by a square cylinder
   - Preliminary results

4 Conclusion and perspectives
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Basic spatial filtering of a given quantity $Q$ is performed as follow:

$$
\langle Q(x) \rangle = Q(x) - \sigma \sum_{j=1}^{D} \sum_{n=-N}^{N} d_n Q(x + n\Delta x_j)
$$

(1)

$0 < \sigma = Cste < 1$ and coefficients $d_n$ depending on the filter stencil.
Adaptive filtering:

The filter coefficient $\sigma$ depends on the shear stress amount:

$$\sigma_a(x) = \sigma_0 \xi(|S|)$$  \hspace{1cm} (2)

$$\xi(|S|) = \left(1 - e^{-\frac{|S(x)|}{S_0}}\right)^2$$  \hspace{1cm} (3)

$|S| = \sqrt{2S_{ij}S_{ij}}$, $S_0$ is a threshold to be determined.
\[ \sigma_a(x) = \sigma_0 \left( 1 - e^{-\left(\frac{|S|}{S_0}\right)^2} \right)^2 \]
\[
\sigma_d(x) = \sigma_0 \left(1 - e^{-\frac{|S|}{S_0}}\right)^2
\]

Threshold estimation:

\(S_0\) could be linked with the maximum amount of shear stress \(S_{\text{max}}\):

\[S_0 = \epsilon S_{\text{max}}\]

- \(S_{\text{max}}\) imposed to a constant.
- \(S_{\text{max}}\) computed from mean field.
- \(S_{\text{max}}\) based on \(g_{\alpha}\) positivity.
LBM shear computation:

From a LBM framework, the shear stress can be obtained with:

\[ 2\rho\nu S_{ij} = -\sum_{\alpha} c_{\alpha,i} c_{\alpha,j} (g_{\alpha} - g_{\alpha}^{eq}) \]  

(4)

Assuming the positivity of \( g_{\alpha} \) and \( \sum_{\alpha} c_{\alpha,i} c_{\alpha,j} g_{\alpha}^{eq} = \rho u_i u_j \)

\[ |S| = \frac{Q_f}{2\rho\nu} = \sqrt{2 \sum_{\alpha} c_{\alpha,i} c_{\alpha,j} g_{\alpha}^{eq} \sum_{\alpha} c_{\alpha,i} c_{\alpha,j} g_{\alpha}^{eq}} \leq \frac{\sqrt{2}u_i u_j}{2\nu} \]

Then:

\[ S_{\text{max}} = \frac{\sqrt{2}U_0^2}{2\nu} \sim \frac{Re_\delta U_0}{\delta} \]
Lattice Boltzmann Method

Lattice Boltzmann algorithm:

\[ g^{\text{coll}}_{\alpha}(x, t) = g_{\alpha}(x, t) - \frac{1}{\tau_g} [g_{\alpha}(x, t) - g^{\text{eq}}_{\alpha}(x, t)] \]

\[ g_{\alpha}(x, t) = g^{\text{coll}}_{\alpha}(x - c_{\alpha} \Delta t, t - \Delta t) \]

\[ \rho = \sum_{\alpha} f_{\alpha} \]

\[ \rho u = \sum_{\alpha} c_{\alpha} f_{\alpha} \]

\[ g^{\text{eq}}_{\alpha}(x, t) = \rho \omega_{\alpha} \Delta \left( 1 + \frac{u \cdot c_{\alpha}}{\tilde{c}_0^2} + \frac{(u \cdot c_{\alpha})^2}{2 \tilde{c}_0^4} - \frac{|u|^2}{2 \tilde{c}_0^2} \right) \]

LBM recovers compressible Navier-Stokes equations in the limit of low Mach Numbers \( O(M^3) \)

\[
\begin{align*}
p & = \tilde{c}_0^2 \rho \\
\tilde{c}_0^2 & = 1/3 \\
\omega_0 & = 1/3 \\
\omega_{1-6} & = 1/18 \\
\omega_{7-18} & = 1/36
\end{align*}
\]
Choosing the filtered quantity

3 different possibilities with increasing computational cost:

1. Filtering momenta: $\rho, u_x, u_y, u_z$ (4 tables).
2. Filtering distribution functions: $g_\alpha$ (19 tables).
3. Filtering collision operator: $-\frac{1}{\tau_g}(g_\alpha - g^{eq}_\alpha)$ (19 tables).
Then the overall algorithm becomes:

**Modified algorithm for moments filtering:**

- Estimation of $S_0$.
- Compute $|S|$ with (4) and update $\sigma_d$
- Collision Step
- (Collision Filtering)
- Propagation Step
- Update moments
- (Moments filtering)
- Update $g^{eq}_{\alpha}$
Towards adaptive relaxation times

MRT Collision

\[ m^*_\alpha = m_\alpha - S (m_\alpha - m^{eq}_\alpha) \]
\[ g_\alpha(x + c_\alpha \Delta t, t + \Delta t) = M^{-1} m^*_\alpha(x, t) \]

In the MRT framework, the filtering step is not needed.

\[
(m_1, \cdots, m_9) = (\rho, e, \epsilon, \rho u_x, q_x, \rho u_y, q_y, p_{xx}, p_{xy})
\]

The function \( \xi(|S|) \) is used to switch from MRT in sheared region to BGK in uniform flow region:

\[
S' = \text{diag} \left( 0, s'_e, s'_\epsilon, 0, s'_q, 0, s'_q, s'_\nu, s'_\nu \right)
\]

where \( s' = s + (1 - \xi)(s_\nu - s) \)
Then the overall algorithm becomes:

**Modified algorithm for adaptive relaxation times:**

- Compute $|S|$ with (4) and update $\xi$
- Update relaxation time matrix $S'$
- Collision Step in the momentum space
- Propagation Step
- Update moments
- Update equilibrium
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Initialization of the macroscopic variables

The initialization of the Taylor-Green vortex is done by setting velocity and pressure in a cubic and fully periodic domain of size $2\pi$:

\[
\begin{align*}
    p &= p_\infty + \frac{\rho_\infty U_\infty^2}{16} \left[ \cos(2z) + 2 \right] \left[ \cos(2x) + \cos(2y) \right] \\
    u &= U_\infty \sin(x) \cos(y) \cos(z) \\
    v &= -U_\infty \cos(x) \sin(y) \cos(z) \\
    w &= 0 
\end{align*}
\]  

(6)

Parameters:
- Imposing $Re = 1600$, $M_\infty = 0.085$, $\rho_\infty = 1$, $dx = 2\pi/n_x$
- Induces: $U_\infty = 0.049$, $p_\infty = 1/3$, $\tau_g = 0.5 + 1.46n_x 10^{-5}$
Initialization of the distribution functions

In order to avoid spurious oscillations from initialization, the distribution functions are initialized with their non-equilibrium part:

\[
\begin{align*}
g^{\text{init}}_\alpha &= g^{\text{eq}}_\alpha + g^{\text{neq}}_\alpha \\
g^{\text{neq}}_\alpha &= -\frac{\omega_\alpha \tau g}{\bar{c}_0^2} \left[ (c_{\alpha,i}c_{\alpha,j} - \bar{c}_0^2 \delta_{ij}) \frac{\partial \rho u_i}{\partial x_j} \right]
\end{align*}
\]

Gradients are evaluated with a centred 2\textsuperscript{nd} order finite difference scheme.

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Dissipation rate of kinetic energy

- Results on a $128^3$ grid
- Adaptive filtering give similar results than the MRT model.
- Adaptive filtering with 3-point stencil gives better results than static 5-point.
- Mode details in Marié and Gloerfelt 2017\textsuperscript{5}

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Case setup

Parameters

- Reynolds $R_D = 150$
- Mach number $M_a = 0.3$
- the square size is $D = 5$
- the domain size is $300D \times 300D$
- The domain is initialized with a uniform flow and periodic boundary conditions.
- A circular sponge zone is defined at the domain boundary to damp the outgoing structures.
Case setup

- Details about LBM sponge zone from Xu and Sagaut 2011\(^6\).
- Here the filter coefficient \(\sigma\) and the relaxation time \(\tau_{\nu}\) are set to high values in the sponge zone.
- For ART model, the switcher is set to 1 in the sponge zone.

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The von-Karmann instability is known to generate sound-waves at a given characteristic frequency: 
\[ S_t = \frac{f \cdot D}{U_\infty} \sim 0.17 \]

### About resolutions
- **Aerodynamic resolution**: \( D \)
- **Acoustic resolution**: \( \lambda = \frac{D}{S_t \cdot M_\infty} \sim 20D \)
- Difficult to handle this resolution in the whole domain.
- Consequently, far field acoustic is often a coarse region where numerical dissipation is high.
- Here, only uniform resolution is considered.
A coustic Validation

Preliminary results

Standard

Adaptive $S_0 = 1.0$

BGK

MRT

S. Marié (DynFluid)
Acoustic Validation
Preliminary results

Standard

Adaptive
$S_0 = 0.5$
Acoustic Validation  Preliminary results

BGK  MRT

Standard

Adaptive  

$S_0 = 0.1$
Maps of $\xi(|S|)$
PSD of the acoustic pressure at a $80D$ distance from the square.
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Conclusion

- The use of adaptive filters with BGK operator give similar results than the MRT model on a 3D TGV dynamics.
- Adaptivity allows to use low stencil filters, consistent with LBM models.
- Adaptivity allows to keep aerodynamic accuracy and keeps the acoustic waves free of numerical dissipation.
- Introducing an adaptive filtering induces an early development of the von Karmann instability.
- Adaptive relaxation times gives promising results for high Reynolds acoustic generation and propagation.
Need for broad-band noise validation to get a wide range of acoustic resolutions. (ie. 3D turbulent jet)

Quantitative comparison to other techniques (Regularization, TRT...)

Focus on the switcher shape (Lighthill source term sensitive switcher)

Take the wall distance into account explicitly (Boundary Layer correction, van Driest...)

Apply adaptive relaxation times approach to a larger range of physical application (multiphase, large scales multicomponent flow...)
Thanks for your attention