Lattice Boltzmann equation with selective viscosity filter

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Computational Aeroacoustics (CAA) and LBM

- CAA = high Reynolds number flow simulation + direct simulation of acoustic fields
- LBM has enough accuracy to simulate acoustic phenomena ([*Buick et al. 1998*, *Ricot et al. 2002*, *Marié et al. 2007*])
- LBM is a low dissipative scheme $\longrightarrow$ unstable in high Reynolds flows (low viscosity)

Stability issue in LBM

- Numerical instability sources : poor initial and boundary conditions, under-resolved shear flow, interpolation errors in multi-resolution simulation...
- Proposed solutions
  - Artifical viscosity (global or local lower bound of the relaxation time [*Li et al., 2004, PowerFLOW*])
  - Dissipative lattice Boltzmann scheme (fractional propagation [*Qian, 1997*])
  - Multiple Relaxation Time model [*Lallemand, 2000*]...
  - ... or increase of the bulk viscosity [*Dellar, 2001*]
  - Explicit filter to damp the high wavenumber oscillations [*Skordos, 1995*]

Presentation outline

- Linearization around a uniform mean flow

$$g_\alpha\left(\mathbf{x}+\mathbf{c}_\alpha, t+1\right)=g_\alpha\left(\mathbf{x}, t\right)-\frac{1}{\tau}\left(g_\alpha\left(\mathbf{x}, t\right)-g_\alpha^{eq}\left(\mathbf{x}, t\right)\right)$$

$$g^{eq}\left(g_\alpha^{(0)}+g_\alpha^{\prime}\right)=g_\alpha^{eq,(0)}+\left.\frac{\partial g_\alpha^{eq}}{\partial g_\beta}\right|_{g_\alpha=g_\alpha^{(0)}} g_\alpha^{\prime}+o\left(\left(g_\alpha^{\prime}\right)^2\right)$$

- Fourier decomposition of the fluctuating distribution functions

$$g_\alpha^{\prime}\left(\mathbf{x}, t\right)=h_\alpha e^{i\left(\mathbf{k}.\mathbf{x}-\omega t\right)}$$

- Eigenvalue / eigenvector problem

$$M\mathbf{h}=e^{-i\omega}\mathbf{h}$$

- Matrix for the LBE-BGK model

$$M^{\mathrm{BGK}} = A^{-1}[I - \frac{1}{\tau} N^{\mathrm{BGK}}]$$

- Matrix for the LBE-MRT model

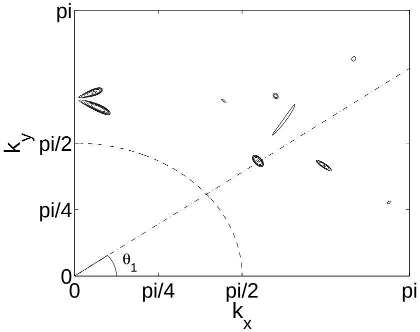
$$M^{\mathrm{MRT}} = A^{-1}[I - P^{-1} S P N^{\mathrm{BGK}}] \quad \text{with} \quad \mathbf{m} = P\mathbf{g} \;, \quad S = diag[\frac{1}{\tau_1}, ..., \frac{1}{\tau_N}]$$

- Link between eigenvalues and macroscopic transport coefficients

$$\begin{cases} Re[\omega^{\pm}(\mathbf{k})] & = & k\left(\pm c_s(\mathbf{k}) + U_0(\mathbf{k})\right) \\ Im[\omega^{\pm}(\mathbf{k})] & = & -k^2\left(\frac{2}{3}\nu(\mathbf{k}) + \frac{1}{2}\eta(\mathbf{k})\right) \end{cases}$$

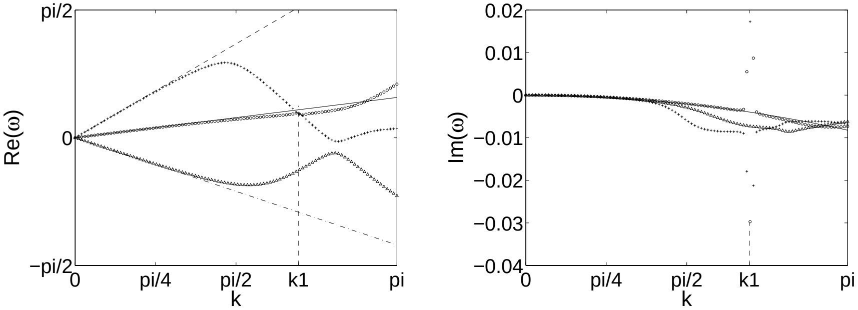
$$\begin{cases} Re[\omega^{T}(\mathbf{k})] & = & kU_0(\mathbf{k}) \\ Im[\omega^{T}(\mathbf{k})] & = & -k^2\nu(\mathbf{k}) \end{cases}$$

- Unstable simulation if $Im[\omega(\mathbf{k})] > 0$
- Stability condition depends on $\mathbf{k}$, $\mathbf{U}_0$, τ



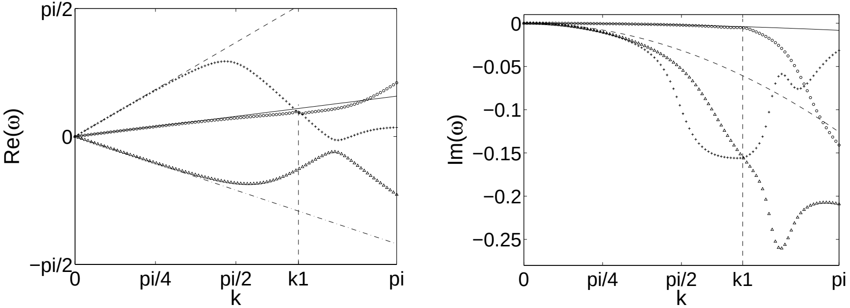
D2Q9-BGK : Isocontours of $Im[\omega(\mathbf{k})] > 0$ (unstable regions) in the wavenumber space (k_x, k_y) for $U_0 = U_x = 0.2$ and $1/\tau = 1.99$.

- D2Q9-BGK : dispersion and dissipation of the three physical modes for $\angle(\mathbf{k}, \mathbf{x}) = \theta_1$
- Dispersion error $\rightarrow$ mode coincidence for $k = k_1$
- "Energy transfer" between the positive acoustic mode and the shear mode



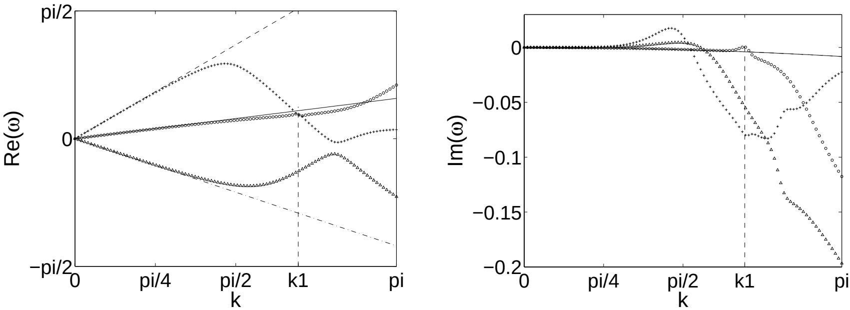
(△), (+) acoustic modes; (○) shear mode; (— — —), (— · —) theoretical dispersion relation of acoustic modes; (———) theoretical dispersion of the shear mode.

- D2Q9-MRT with standard relaxation times (*[Lallemand & Luo, 2000]*) : "bulk viscosity" relaxation time $1/\tau_2 = 1.64$, "shear viscosity" relaxation times $1/\tau_8 = 1/\tau_9 = 1.99$
- The dispersion error is the same as D2Q9-BGK
- Mode coincidence occurs but $Im[\omega(\mathbf{k})]$ remains negative



(△), (+) acoustic modes; (○) shear mode; (— — —), (— · — ·) theoretical dispersion relation of acoustic modes; (———) theoretical dispersion of the shear mode.

- D2Q9-MRT with the same bulk viscosity as BGK model ($1/\tau_2 = 1.99$)
- In this case the MRT model is unstable
- Other undamped interactions between acoustic modes and kinetic modes occur around $k \approx \pi/2$



(△), (+) acoustic modes; (○) shear mode; (— — —), (— · — ·) theoretical dispersion relation of acoustic modes; (———) theoretical dispersion of the shear mode.

Conclusion on the stability analysis

- Numerical instabilities are due to "energy transfer" between acoustic modes and and the other modes
- Standard MRT model is stable but high bulk viscosity must be used
- Standard BGK model : mode interactions occur in high wavenumber domain

$\longrightarrow$ selective wavenumber filter to damp high wavenumber only

- Genereral expression of the filtering operator $\langle\rangle$ for a given variable v :

$$\langle v(\mathbf{x})\rangle = v(\mathbf{x}) - \sigma \sum_{j=1}^{D} \sum_{n=-N}^{N} d_n v(\mathbf{x} + n\mathbf{x}_j)$$

  D : space dimension ($D = 2$ in this work)

  $2N + 1$: number of points of the damping stencil

  $0 < \sigma < 1$: strength of the filter

- Filters used in this study :
  - Standard 5-point stencil (tested by *[Skordos, 1995]*)
  - Standard 7-point stencil
  - Optimized 7-point stencil (*[Tam et al., 1993]*)
  - Optimized 9-point stencil (*[Bogey & Bailly, 2004]*)

First approach

- The filtering operator is applied to the distribution functions

$$\begin{cases} g_\alpha(\mathbf{x},t) = \langle g_\alpha(\mathbf{x}-\mathbf{c}_\alpha,t-1)\rangle - \frac{1}{\tau}\left(\langle g_\alpha(\mathbf{x}-\mathbf{c}_\alpha,t-1)\rangle - g_\alpha^{\langle eq\rangle}(\mathbf{x}-\mathbf{c}_\alpha,t-1)\right) \\ g_\alpha(\mathbf{x},t) \rightarrow \langle g_\alpha(\mathbf{x},t)\rangle \end{cases}$$

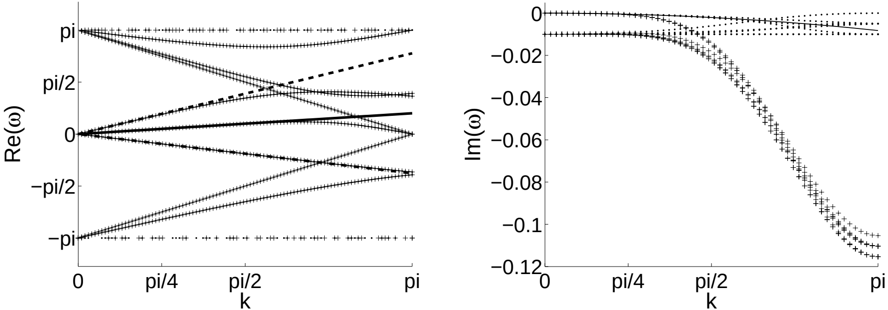
- New matrix of the eigenvalue problem

$$M_{\langle g_\alpha\rangle} = (1-\sigma f)A^{-1}[I - \frac{1}{\tau}N^{\mathrm{BGK}}]$$

with the filter function f defined as :

$$f(\mathbf{k}) = \sum_j \sum_n d_n e^{in\mathbf{k}\cdot\mathbf{x}_j}$$

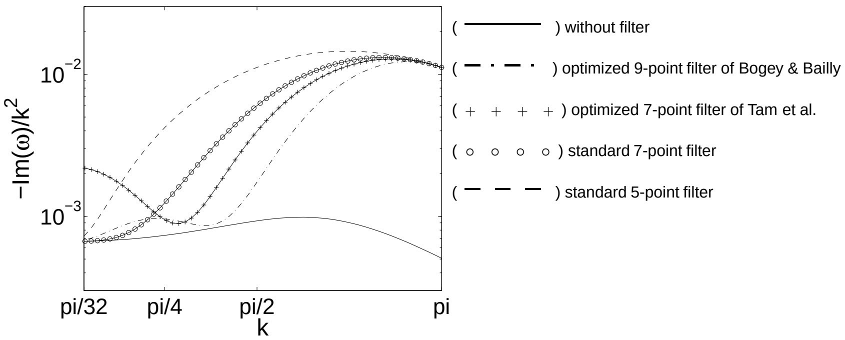
- Dispersion and dissipation of the D2Q9-BGK filtered with the standard 7-point filter



(· · · · · ·) without filter, (+) with filter;
(— — —), (— · — ·) theoretical dispersion relation of acoustic modes;
(———) theoretical dispersion of the shear mode.

- $d_{-n} = d_n \rightarrow$ the filter does not introduce dispersion error

- Effective viscosity can be defined as $-Im[\omega(\mathbf{k})]/k^2$
- Comparison of the effective bulk viscosity for the various filters :



Second approach

- Filtered distribution functions imply filtered macroscopic variables
- New algorithm : the filtering operator is only applied to macroscopic variables

$$\begin{cases} g_\alpha(\mathbf{x},t) = g_\alpha(\mathbf{x}-\mathbf{c}_\alpha,t-1) - \frac{1}{\tau}\left(g_\alpha(\mathbf{x}-\mathbf{c}_\alpha,t-1) - g_\alpha^{\langle eq\rangle}(\mathbf{x}-\mathbf{c}_\alpha,t-1)\right) \\ \rho(\mathbf{x},t) \rightarrow \langle\rho(\mathbf{x},t)\rangle \\ \rho u_j(\mathbf{x},t) \rightarrow \langle\rho u_j(\mathbf{x},t)\rangle \end{cases}$$

- New matrix of the eigenvalue problem

$$M_{g_\alpha^{\langle eq\rangle}} = A^{-1}[I - \frac{1}{\tau}\left(I - (1-\sigma f)\,G^{eq}\right)]$$

Third approach

- Numerical instabilities are often generated in regions where the nonequilibrium parts g_α^{neq} of distribution functions become (too) large
- A third filtering strategy is based on a filtered collision operator

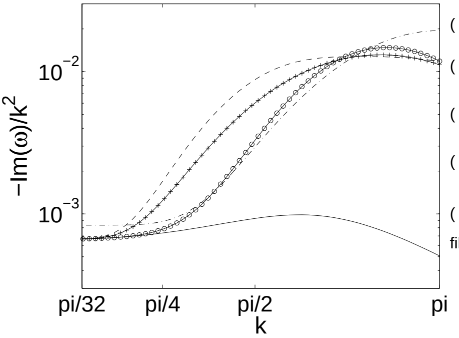
$$\begin{cases} g_\alpha^{neq}(\mathbf{x},t) \rightarrow \langle g_\alpha^{neq}(\mathbf{x},t)\rangle \\ \langle g_\alpha(\mathbf{x}+\mathbf{c}_\alpha, t+1)\rangle_{coll} = g_\alpha(\mathbf{x},t) - \frac{1}{\tau}\langle g_\alpha^{neq}(\mathbf{x},t)\rangle \end{cases}$$

- New matrix of the eigenvalue problem

$$M_{g_\alpha^{\langle coll \rangle}} = A^{-1}[I - \frac{(1-\sigma f)}{\tau} N^{\mathrm{BGK}}]$$

Comparison of the three filtering strategies

- Comparison of the effective bulk viscosity for the three filtering approaches
- Only results obtained with the standard 7-point filter are shown but conclusions are the same for other stencils



(———) without filter

(+ + + +) fully filtered LBE $\langle g_\alpha \rangle$

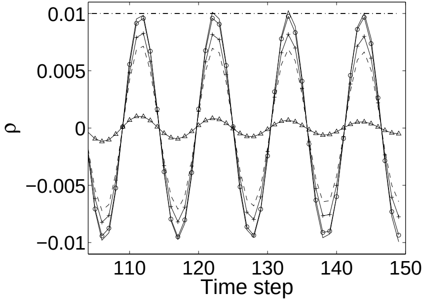
(— — —) filtered macroscopic variables $g_\alpha^{\langle eq \rangle}$

(○ ○ ○ ○) filtered collision operator $g_\alpha^{\langle coll \rangle}$

(— · — ·) theoretical effective bulk viscosity of the filtered collision operator model :

$$\nu = c_s^2 \left(\frac{\tau}{1 - \sigma f(k)} - \frac{1}{2} \right)$$

- Propagation of a plane acoustic wave with $k_a = \pi/3$ (6 points per wavelength) in a periodic domain; $U_0 = 0$, $1/\tau = 1.99995$, $\sigma = 0.1$
- Time signal after propagation over 10 wavelengths :



(———) without filter

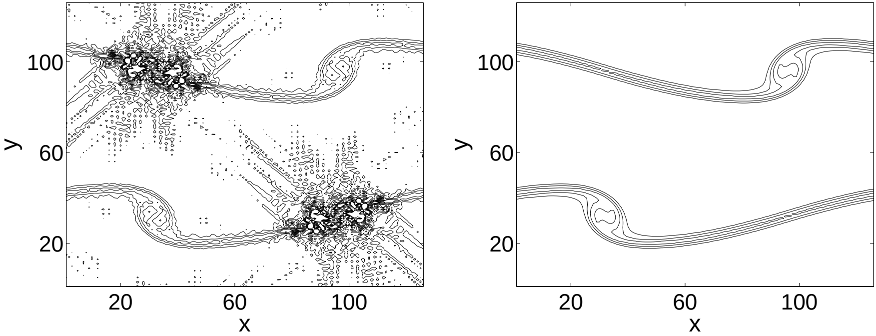
(○ ○ ○ ○) filtered collision operator $g_\alpha^{\langle coll\rangle}$

(+ + + +) fully filtered LBE $\langle g_\alpha \rangle$

(— — —) filtered macroscopic variables $g_\alpha^{\langle eq\rangle}$

(△ △ △) MRT

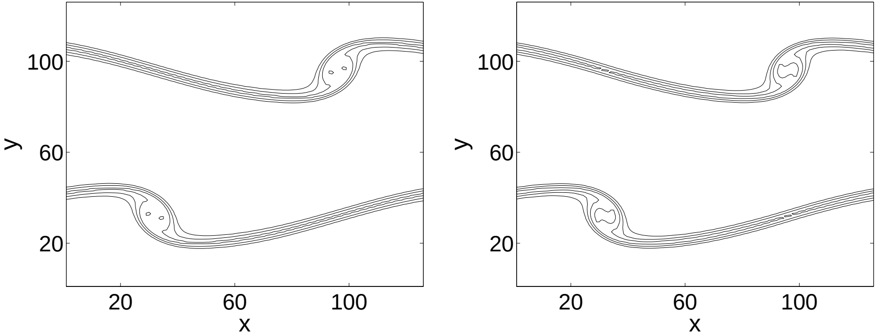
- Doubly periodic shear layer with initial perturbation
- About 9 points across shear layers
- 128×128 grid, $1/\tau = 1/\tau_8 = 1/\tau_9 = 1.9988$, $\sigma = 0.01$



Standard BGK

MRT ($1/\tau_2 = 1.64$)

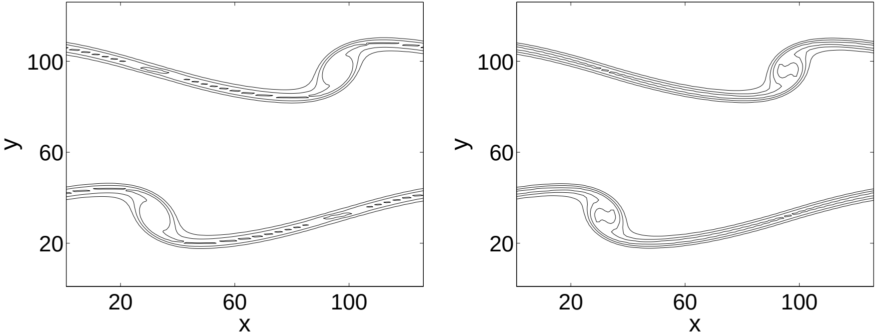
- Doubly periodic shear layer with initial perturbation
- About 9 points across shear layers
- 128×128 grid, $1/\tau = 1/\tau_8 = 1/\tau_9 = 1.9988$, $\sigma = 0.01$



Fully filtered BGK

MRT ($1/\tau_2 = 1.64$)

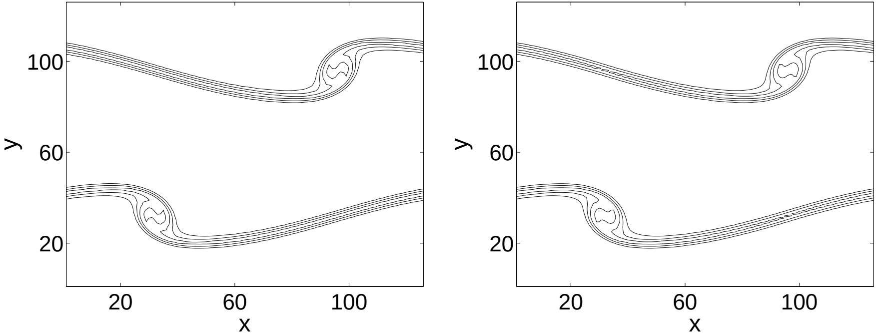
- Doubly periodic shear layer with initial perturbation
- About 9 points across shear layers
- 128×128 grid, $1/\tau = 1/\tau_8 = 1/\tau_9 = 1.9988$, $\sigma = 0.01$



Filtered macroscopic variables

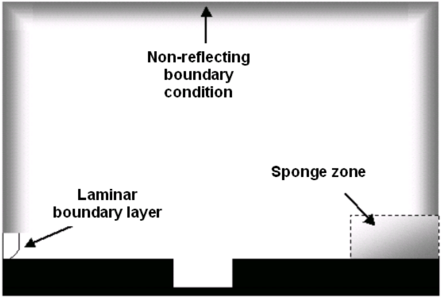
MRT ($1/\tau_2 = 1.64$)

- Doubly periodic shear layer with initial perturbation
- About 9 points across shear layers
- 128×128 grid, $1/\tau = 1/\tau_8 = 1/\tau_9 = 1.9988$, $\sigma = 0.01$



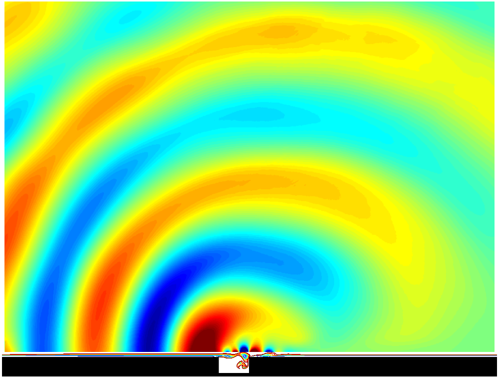
Filtered collision operator — MRT ($1/\tau_2 = 1.64$)

- Self-sustained oscillation of flow over rectangular cavity
- *Mach* $= 0.25$, $1/\tau = 1.98$, $L/\theta_0 = 52$ (θ_0 : boundary layer momentum thickness)
- Unstable simulation without selective viscosity filter



Simulation setup

- Example of simulation with filtered macroscopic variables ($\sigma = 0.15$)



Snapshot of vorticity and acoustic pressure

- Results are in good qualitative agreement with other CAA simulations (*[Gloerfelt et al. 2001, Rowley at al. 2002]*)

Conclusion

- Selective filters damp unphysical instabilities without affecting physical waves
- Increase of the computational effort
  - lost of "locality" : sharper cut-off filter at higher wavenumber needs more far points
  - macroscopic variable filtering is the less expensive approach
  - it is not necessary to apply the filter at each time step
  - it is not necessary to apply the filter in the whole computational domain
- The best efficiency is obtained with the filtered collision operator : a wavenumber-dependent viscosity is obtained
- Explicit filtering is well suited for LES subgrid models